A mistake in finding minimum value in quadratic function

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Question

(a) Find the maximum value of $1-4x-4x^2$.

(b) Hence deduce the minimum value of $2-\dfrac{3}{4x^2+4x-1}$.

"Solution" (taken from New Way Mathematics Book 1 P.70, Practice 19 Teacher Edition)

(a) $1-4x-4x^2=-4\left(x^2+x-\frac{1}{4}\right)=-4\left[x^2+x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-\frac{1}{4}\right]=-4\left[\left(x+\frac{1}{2}\right)^2-\frac{1}{2}\right]=-4\left(x+\frac{1}{2}\right)^2+2$

Since $-4\left(x+\frac{1}{2}\right)^2\le 0$ for all real values of x, $1-4x-4x^2$ has the max. value of 2, when $x=-\frac{1}{2}$.

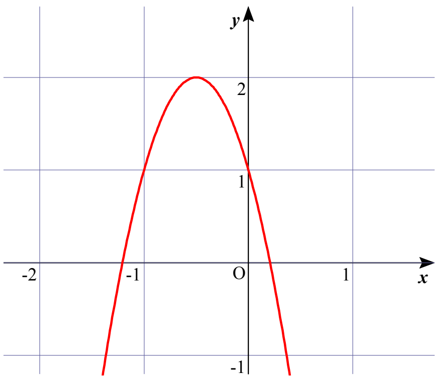
(b) $2-\dfrac{3}{4x^2+4x-1}=2+\dfrac{3}{1-4x-4x^2}$ has the minimum value when the denominator of $\dfrac{3}{1-4x-4x^2}$ has a maximum value 2.

$\therefore$ The minimum value $= 2+\dfrac{3}{2}=\underline{\underline{\dfrac{7}{2}}}$

Analysis

$f(x)=1-4x-4x^2$ of has a max. of 2 does not imply $g(x)=\dfrac{1}{1-4x-4x^2}$ has a min. value of $\dfrac{1}{2}$.

Let the roots of $f(x)=0$ be $\alpha=\dfrac{-1-\sqrt{2}}{2}$, $\beta=\dfrac{-1+\sqrt{2}}{2}$.



It can be seen that from the diagram on the right,

(1) $f(x)\ge 0$ if $\alpha\le x\le\beta$.

(2) $f(x)<0$ if $x<\alpha$ or $x>\beta$.

Since $g(x)=\dfrac{1}{f(x)}$,

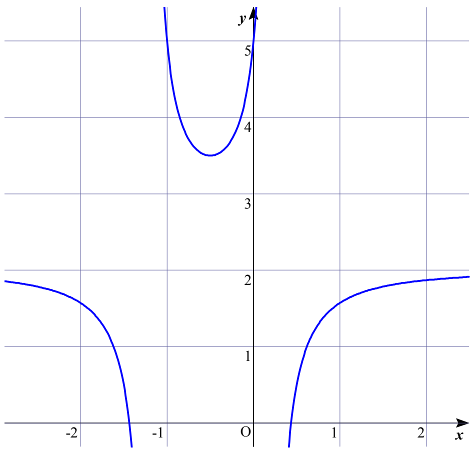
(1) $g(x)>0$ if $\alpha<x<\beta$.

(2) $g(x)<0$ if $x<\alpha$ or $x>\beta$.

(3) $g(x)$ is undefined if $x=\alpha$ or $x=\beta$.

$g(x)$ has only a local min. of $\frac{1}{2}$ when $x=-\frac{1}{2}$.

$g(x)$ can be any negative value if $x<\alpha$ or $x>\beta$.



The graph of $h(x)=2-\dfrac{3}{4x^2+4x-1}$ is shown on the right diagram.

(1) $h(x)$ has a local min. of $\frac{7}{2}$ when $x=-\frac{1}{2}$.

(2) $h(x)<2$ if $x<\alpha$ or $x>\beta$.